11.21 交第七次作业答案.

(一) 选择题

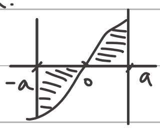
1. B 定义

2. A. $\ln|x| + C = \ln e^{C}\cdot|x| = \ln|ax|$

3. B. $\int f'\,dx = f(x)+C = (\sin x)'+C = \cos x + C.$

4. D. $(\cosh^2 x)' = 2\cosh x \sinh x = \sinh 2x.$

5. D. D项 $=0$



6 B. $\int df(x) = \int f'(x)dx = f(x)$

7. A. 积分与求导需乘的系数互为倒数.

8. D. $\int \frac{1}{x}dx = \ln|x|.$

9 D. $\int x f(1-x^2)dx = -\frac{1}{2}\int f(1-x^2)d(1-x^2) = -\frac{1}{2}(1-x^2)^2+C.$

(二) 解答题

3. (1) $\int x^4 dx = \frac{1}{5}x^5 + C.$

(2) $\int x\sqrt{x}\,dx = \int x^{3/2}dx = \frac{2}{5}x^{5/2}+C$

(3) $\int (\frac{1}{x}+4^x)dx = \ln|x| + \frac{1}{\ln 4}4^x + C$

(4) $\int \frac{x^2-2\sqrt{2}x+2}{x-\sqrt{2}}dx = \int x-\sqrt{2}\,dx = \frac{1}{2}x^2-\sqrt{2}x + C$

(5) $\int \tan^2 x\,dx = \int \frac{1}{\cos^2 x} - 1\,dx = \tan x - x + C$

(6) $\int \frac{2x^2+3}{x^2+1}dx = \int 2+\frac{1}{x^2+1}dx = 2x + \arctan x + C.$

(7) $\int \frac{\cos 2x}{\cos x - \sin x}dx = \int \frac{\cos^2 x - \sin^2 x}{\cos x - \sin x}dx = \int \cos x + \sin x\,dx = \sin x - \cos x + C.$

(8) $\int (1+\cos^3 x)\sec^2 x\,dx = \int \sec^2 x + \cos x\,dx = \tan x + \sin x + C.$

4. (1). 令 $t=\sqrt{2+3x}$ $x=\frac{1}{3}(t^2-2)$, $dx = \frac{2}{3}t\,dt$

$\int \sqrt{2+3x}\,dx = \int t \cdot \frac{2}{3} t\,dt = \int \frac{2}{3} t^2 dt = \frac{2}{9} t^3 + C = \frac{2}{9}(2+3x)^{3/2} + C.$

(2) 令 $t = 1-2x$. $dt = -2dx \Rightarrow dx = -\frac{1}{2}dt$

$\int \frac{4}{(1-2x)^2}dx = \int 4t^{-2} \cdot -\frac{1}{2}dt = \int -2t^{-2}dt = \frac{2}{t} + C = \frac{2}{1-2x} + C$

(3) 令 $t = x^2$. $dt = 2x\,dx$

$\int x\sqrt{x^2+3}\,dx = \int \sqrt{t+3} \cdot \frac{1}{2}dt = \frac{1}{3}(t+3)^{3/2} + C = \frac{1}{3}(x^2+3)^{3/2} + C.$

(4) 令 $t = 3x$. $dx = \frac{1}{3}dt$.

$\int \sin 3x\,dx = \int \frac{1}{3}\sin t\,dt = -\frac{1}{3}\cos t + C = -\frac{1}{3}\cos 3x + C.$

(5) 令 $x = \frac{1}{5}\sin t$. $dx = \frac{1}{5}\cos t\,dt$ $-\pi/2 < t < \pi/2$, $\cos t > 0$

$\int \frac{1}{\sqrt{1-25x^2}}dx = \int \frac{1}{|\cos t|} \cdot \frac{1}{5}\cos t\,dt = \frac{1}{5}t + C = \frac{1}{5}\arcsin 5x + C.$

(6). 令 $x = \frac{1}{3}\tan t$. $dx = \frac{1}{3}\sec^2 t$.

$\int \frac{1}{1+9x^2}dx = \int \frac{1}{3}\cos^2 t \sec^2 t\,dt = \frac{1}{3}t + C = \frac{1}{3}\arctan 3x + C.$

(7) 令 $t = 2x$. $dx = \frac{1}{2}dt$

$\int \cos^2 x\,dx = \int \frac{\cos 2x + 1}{2}dx = \frac{1}{2}x + \frac{1}{2}\int \cos t \cdot \frac{1}{2}dt = \frac{1}{2}x + \frac{1}{4}\sin 2x + C.$

(8) 令 $t = e^x$. $dt = t\,dx$

$\int \frac{e^x}{2-3e^x}dx = \int \frac{1}{2-3t}dt = -\frac{1}{3}\ln|2-3t| + C = -\frac{1}{3}\ln|3e^x - 2| + C.$

熟练后省略:

(9) $\int e^x \cdot (e^x+2)^5 dx = \int (e^x+2)^5 de^x = \frac{1}{6}(e^x+2)^6 + C.$

(10). $\int \frac{1}{x^2+2x+3}dx = \int \frac{1}{(x+1)^2+2}d(x+1) = \frac{\sqrt{2}}{2}\int \frac{1}{1+[\frac{\sqrt{2}}{2}(x+1)]^2}d(\frac{\sqrt{2}}{2}(x+1))$

$= \frac{\sqrt{2}}{2}\arctan[\frac{\sqrt{2}}{2}(x+1)] + C.$

(11). $\int \frac{1}{x^2-16}dx = \int \frac{1}{8}(\frac{1}{x-4} - \frac{1}{x+4})dx = \frac{1}{8}\ln\left|\frac{x-4}{x+4}\right| + C.$

(12). $\int 10^{2x}dx = \frac{1}{2\ln 10} \cdot 10^{2x}dx$

(13) $\int \sin 3x \cdot \sin 5x\,dx = \frac{1}{2}\int(\cos 2x - \cos 8x)dx = \frac{1}{4}\sin 2x - \frac{1}{16}\sin 8x + C$

(14) $\int \cos^3 x\,dx = \int (1-\sin^2 x)\,d\sin x = \sin x - \frac{1}{3}\sin^3 x + C.$

(15) $\int \frac{1}{\sqrt{4-9x^2}} dx = \frac{1}{2}\int \frac{1}{\sqrt{1-t^2}} d(\frac{2}{3}t) = \frac{1}{3}\arcsin\frac{3}{2}x + C$

(16) $\int \frac{1}{x\sqrt{x^2+1}} dx = \int \frac{1}{(\sqrt{x^2+1})^2-1} d\sqrt{x^2+1} = \int \frac{1}{2}(\frac{1}{t-1} - \frac{1}{t+1}) dt = \frac{1}{2}\ln\left|\frac{t-1}{t+1}\right| + C$

$= \ln\left|\frac{x}{\sqrt{x^2+1}+1}\right| + C$

(17). $\int \frac{\sqrt{x^2-a^2}}{x} dx = \int \frac{x^2-a^2}{x^2} d\sqrt{x^2-a^2} = \int \frac{t^2}{t^2+a^2} dt = t - a\cdot\arctan t/a + C$

$= \sqrt{x^2-a^2} - a\cdot\arctan\sqrt{\frac{x^2-a^2}{a^2}} + C$

形式不唯一.