10.24交第四次作业答案

1 (1). $y' = 2x$ (2) $y' = -\sin(x+2)$.

2. $\lim\limits_{x\to x_0} \dfrac{f(x)-f(x_0)}{x-x_0} = f'(x_0)$

3. (1). $\lim\limits_{\Delta x\to 0} \dfrac{f(x_0)-f(x_0-\Delta x)}{\Delta x} = \lim\limits_{-\Delta x\to 0} \dfrac{f(x_0-\Delta x)-f(x_0)}{-\Delta x} = f'(x_0)$. 正确.

(2) $\lim\limits_{\Delta x\to 0} \dfrac{f(x_0+\Delta x)-f(x_0-\Delta x)}{2\Delta x} = \dfrac{1}{2}\lim\limits_{\Delta x\to 0}\left(\dfrac{f(x_0+\Delta x)-f(x_0)}{\Delta x} + \dfrac{f(x_0)-f(x_0-\Delta x)}{\Delta x}\right)$

[$f(x)$可以不连续.] $= f'(x_0)$. (若$f'(x_0)$存在). 错误.

4 $f'(0) = \lim\limits_{x\to 0} \dfrac{f(x)-f(0)}{x-0} = \lim\limits_{x\to 0} x^{k-1}\sin\dfrac{1}{x}$.
$\begin{cases} 在k=2时可导 \\ k=0,1时不可导. \end{cases}$

5. $f'_+(0) = \lim\limits_{x\to 0^+} \dfrac{|\sin x|-|\sin 0|}{x-0} = 1$. $f'_+(0) \neq f'_-(0)$. 不存在导数.

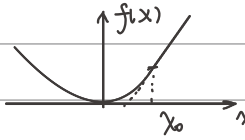
$f'_-(0) = \lim\limits_{x\to 0^-} \dfrac{|\sin x|-|\sin 0|}{x-0} = -1$

6. $f(x)$ 可导性仅在 x_0 处有危险, $f'_-(x_0) = 2x_0$; $f'_+(x_0) = a$

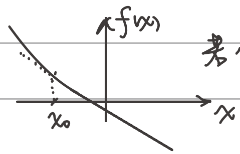
$f'_+(x_0) = f'_-(x_0) \Rightarrow a = 2x_0$.

仍需 f 连续, $f_-(x_0) = x_0^2$, $f_+(x_0) = ax_0+b = 2x_0^2+b$

$f_+(x_0) = f_-(x_0) \Rightarrow b = -x_0^2$. f草图如下:



若 $x_0>0$.



若 $x_0<0$

7 (1) $y' = \left(1+\dfrac{2}{x-1}\right)' = -\dfrac{2}{(x-1)^2}$ (2). $y' = 30x^2+4x-15$

(3) $y' = (x+1)e^x$ (4) $y' = \dfrac{\sin x}{\cos^2 x}$

(5) $y' = \dfrac{-4x}{(x^2-1)^2}$ (6) $y' = ((x-1)^{20})' = 20(x-1)^{19}$

(7) $y' = 3\cos x - 2\cos x\sin x$ (8) $y' = \dfrac{1}{(x^2+1)^2}\left(\dfrac{x^2+1}{\cos x} - 2x\tan x\right)$

(9) $y' = 4\cos 4x$ (10) $y' = (e^{6x\ln 10})' = 6\ln 10\cdot 10^{6x}$

(11) $y' = \left(\dfrac{x^2}{2}+2x+\dfrac{1}{2}\right)e^{\frac{x}{2}}$ (12) $y' = \dfrac{2}{\sqrt{1-(2x+3)^2}}$

(13) $y' = \frac{\cos x}{\sin x} = \cot x$ (14). $y' = \frac{3\ln^2 x}{x}$.

(15) $y' = \frac{1}{x^2+2} \cdot \frac{x}{\sqrt{x^2+1}}$ (16) $y' = \frac{1}{\sqrt{1-(1/x)^2}} \cdot -\frac{1}{x^2} = -\frac{1}{|x|\sqrt{x^2-1}}$

(17). $y' = \frac{1}{x+\sqrt{x^2+a^2}}\left(1+\frac{x}{\sqrt{x^2+a^2}}\right) = \frac{1}{\sqrt{x^2+a^2}}$ 反双曲正弦.

(18). $y' = [\exp(\frac{1}{x}\ln x)]' = x^{\frac{1}{x}}\left(\frac{1}{x^2}(1-\ln x)\right) = x^{\frac{1}{x}-2}(1-\ln x)$

(19) $y' = [\exp(\cos x \ln \sin x)]' = \sin x^{\cos x} \cdot \left(-\sin x \cdot \ln \sin x + \frac{\cos^2 x}{\sin x}\right)$

(20). $\left(\sqrt{\frac{A}{B}}\right)' = \frac{1}{2}\sqrt{\frac{A}{B}} \cdot \left(\frac{A'}{A} - \frac{B'}{B}\right)$

$y' = \frac{1}{2}\sqrt{\frac{x-1}{x(x+3)}}\left(\frac{1}{x-1} - \frac{2x+3}{x(x+3)}\right)$

$= \frac{1}{2}\sqrt{\frac{x-1}{x(x+3)}}\left(\frac{1}{x-1} - \frac{1}{x} - \frac{1}{x+3}\right)$.

8. $y'_-(0) = 2$, $y'_+(0) = \frac{1}{1+x}\Big|_{x=0} = 1$ 从而 $y'(0)$ 不存在.

9 $f'_x = \begin{cases} -2e^{2x} & x<0 \\ 2x & x>0. \end{cases}$ $x=0$ 时不存在.

10. $f_+(1) = 5$. $f_-(1) = 2$. 不连续 $\Rightarrow$ f 在 $x=1$ 不可导.

11. 原式$^{-1}$ $= \lim_{x\to 0} \frac{f(x_0-2x) - f(x_0-x)}{x} = -f'(x_0) = 1$ 原式 $= \frac{1}{1} = 1$.

12. $F'(0) = f'[g(0)]\, g'(0) = f'(0) \cdot \lim_{x\to 0} \frac{x^3\sin\frac{1}{x} - 0}{x-0} = f'(0)\, 0 = 0$.

13. 两边同时求导，有

(1). $2(x-2) + 2(y-3)\cdot y' = 0$ $\Rightarrow y' = -\frac{x-2}{y-3}$

(2). $-\sin(xy)(y+xy') = 1$. $\Rightarrow y' = \frac{1}{x}\left(-\frac{1}{\sin(xy)} - y\right)$

(3). $y' = e^y + xy'e^y$ $\Rightarrow y' = \frac{e^y}{1-xe^y}$

(4) $x^y = y^x \Leftrightarrow y\ln x = x\ln y \Leftrightarrow y'\cdot\ln x + \frac{y}{x} = \ln y + \frac{x}{y}\cdot y'$

得 $y' = \frac{\ln y - y/x}{\ln x - x/y}$.

14. $x^2+3xy+y^2+1=0$ 求导有 $2x+3y+3xy'+2yy'=0$. 在 $(2,-1)$ 处代入 得 $4y'+1=0$ 即 $y' = -\frac{1}{4}$ 切线方程 $y = -\frac{1}{4}x - \frac{1}{2}$.

法线方程 $y = 4x-9$.

15. (1). $y' = \frac{1}{(x+1)^4}\left((x+1)^2 - 2(x+1)(x-1)\right) = \frac{-x+3}{(x+1)^3}$

$y'' = (x+1)^{-6}\left(-(x+1)^3 + 3(x-3)\cdot(x+1)^2\right)$

$= (x+1)^{-4}(2x-10)$

(2) $y' = (1+2x^2)e^{x^2}$. $y'' = (4x^3+6x)e^{x^2}$

(3). $y' = (\cos x - \sin x)\cdot e^x$ $y'' = -2\sin x\, e^x$.

(4). $y' = \cot x$. $y'' = -\frac{1}{\sin^2 x}$

16. $e^{2y} = \frac{1-x}{1+x^2}$. 求导得 $2e^{2y}\cdot y' = (1+x^2)^{-2}\left((x-1)(2x) - (1+x^2)\right)$

$= \frac{x^2-2x-1}{(x^2+1)^2} = \frac{1}{x^2+1} - \frac{2(x+1)}{(x^2+1)^2}$

代入 $x=0$. $y|_0 = 0$ 得 $y' = -\frac{1}{2}$.

再求导: $(2y''+4y'^2)e^{2y} = \frac{-2x}{(x^2+1)^2} - \frac{2}{(x^2+1)^4}\left((x^2+1)^2 - 4x(x+1)(x^2+1)\right)$

$= \frac{-2x}{(x^2+1)^2} - \frac{2}{(x^2+1)^3}(-3x^2-4x+1)$

代入 $x=0$. $y|_0=0$ $y'|_0 = -\frac{1}{2}$ 有 $2y''+1 = -2$ 得 $y''|_0 = -\frac{3}{2}$

17 $y' = \frac{1}{\sqrt{1+x^2}}$; $y'' = -\frac{1}{2}(1+x^2)^{-\frac{3}{2}}\cdot 2x = -x(1+x^2)^{-\frac{3}{2}}$

$y''' = -(1+x^2)^{-\frac{3}{2}} + \frac{3}{2}x(1+x^2)^{-\frac{5}{2}}\cdot 2x. = (1+x^2)^{-\frac{3}{2}}\left(\frac{3x^2}{1+x^2}-1\right)$

代入 $x=\sqrt{3}$ 有 $y'''|_{\sqrt{3}} = \frac{1}{8}\left(\frac{5}{4}\right) = \frac{5}{32}$.

18. $\frac{d}{dx}$: $1+2y'+\sin y\cdot y' = 0$. 得 $\frac{dy}{dx} = y' = -\frac{1}{2+\sin y}$

$\frac{d^2}{dx^2}$: $2y'' + \cos y\cdot(y')^2 + \sin y\cdot y'' = 0$ 得 $\frac{d^2y}{dx^2} = y'' = \frac{-\cos y\ y'^2}{2+\sin y}$

代入有 $\frac{d^2y}{dx^2} = -\frac{\cos y}{(2+\sin y)^3}$.

19 $y' = \frac{-1}{2\sqrt{x}}e^{-\sqrt{x}} + \frac{1}{2\sqrt{x}}e^{\sqrt{x}} = \frac{1}{2\sqrt{x}}(e^{\sqrt{x}} - e^{-\sqrt{x}})$. 则 $2\sqrt{x}y' = e^{\sqrt{x}} - e^{-\sqrt{x}}$

有. $(2\sqrt{x}y')' = \frac{1}{2\sqrt{x}}e^{\sqrt{x}} + \frac{1}{2\sqrt{x}}e^{-\sqrt{x}} = \frac{1}{2\sqrt{x}}y$.

而: $(2\sqrt{x}y')' = \frac{1}{\sqrt{x}}y' + 2\sqrt{x}y''$

于是 $2\sqrt{x}y'' + \frac{1}{\sqrt{x}}y' = \frac{1}{2\sqrt{x}}y$.

即 $xy'' + \frac{1}{2}y' - \frac{1}{4}y = 0$

20. $f(x)=\sum_{k=0}^{n} x^k \quad g(x)=\sum_{k=1}^{n} kx^{k-1}$. 则 $f(x)=\frac{1-x^{n+1}}{1-x} \quad g(x)=f'(x)$

$\Rightarrow g(x)=(1-x)^{-2}\cdot\left((n+1)x^n(x-1)-(x^{n+1}-1)\right)$

$=(1-x)^{-2}\left(nx^{n+1}-(n+1)x^n+1\right)$

21. $\Delta y = y(2+\Delta x)-y(2)$, $dy = y'|_{x=2}\cdot \Delta x$

$\Delta x=1$: $\Delta y = 19$ $dy=12$.

$\Delta x=0.1$. $\Delta y=1.261$. $dy=1.2$

$\Delta x=0.01$: $\Delta y=0.120601$. $dy=0.12$

22. (1) $dy=-\frac{1}{x^3}dx$. (2) $dy=\sin 2x\,dx$

(3). $dy=(x+1)e^x dx$ (4). $dy=5x^{5x}\cdot(\ln x+1)\,dx$.

23. (1) $\Delta x=0.01$. $e^{1.01}\approx e^1+e^1\cdot\Delta x=2.7455$

(2). $\Delta x=1°$. $\cos 151° \approx \cos\frac{5}{6}\pi-\sin\frac{5}{6}\pi\cdot\Delta x$

$=-\frac{\sqrt{3}}{2}-\frac{1}{2}\cdot\frac{\pi}{180}=-0.8747$

24 $S=4\pi r^2$. $dS=S'_r\cdot\Delta r=8\pi r\Delta r$.